PROJECT REPORT

For
Nuclear Engineering Course on
Radiation Protection and Shielding

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The Ohio State University

Project Title:

Group 1: Co-60 Volume Source Shielding

Student Name(s):

Robert Barr
Jake Buschelmann
Lauren King
Colin Voorhis

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Instructor(s):

Praneeth Kandlakunta, Ph.D.
Rich Denning, Ph.D.

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Abstract

This project considers the radiation protection and shielding design of a Cobalt-60 volume source. The main objective is to design the encapsulation layers surrounding this Cobalt-60 source and to design a shielding system which reduces the radiation exposure from the source to within an acceptable dose rate at a specified distance. The source was first modelled mathematically with encapsulation layers only, then analyzed using Monte Carlo N-Particle (MCNP) simulations to validate the veracity of the mathematical model and provide additional insight into factors not considered in our equations. These simulations are run for two combinations of encapsulation materials to determine which is more effective. Once verified, these mathematical models are then extended to analyze various combinations of shielding geometry to determine what materials, in what order, can most effectively reduce the dose rate from this source to 0.1 mSv/hr at a distance of 2 meters. In the following report, this process is explained in further detail and used to confirm that for a single material shield, lead is the most effective material, and a combination of lead and concrete can be used to further optimize the shielding design with concern to size, mass, and cost.

1. Introduction

This project is intended to address the encapsulation and shielding techniques for an example radiation source and develop an understanding of Monte-Carlo simulations via MCNP and the mathematics used to analyze nuclear shielding. The radiation source in question is a uniform cylindrical sample of Cobalt-60. The Cobalt-60 source is encapsulated in a set of three concentric cylindrical protective layers to shield it from mechanical damage and prevent contamination. The inner layer consists of stainless steel (SS) 304, intended to provide structural support and basic containment for the radioactive source without reacting with the source. The middle layer can consist of either ceramic (alumina, Al_2O_3) or graphite. One goal of this project is to determine which material serves as a better encapsulation layer, so the selection of the material for this layer will be assessed in the results and analysis section. This layer's purpose is to provide thermal insulation and corrosion resistance to protect the source from environmental damage. The final encapsulating layer is made of lead, which is intended to provide primary radiation shielding and further physical protection for the source. This encapsulation and source geometry is analyzed both mathematically and using MCNP to determine which middle encapsulation material is more effective. The use of both techniques also allows MCNP to be used to verify the mathematical models employed, allowing them to be used for more advanced shielding analysis after simulations are complete.

These encapsulation layers are further surrounded by an additional layer or layers of shielding materials, including lead, stainless steel, and concrete. The purpose of these layers is solely to reduce the absorbed dose in air outside of the encapsulation and shielding layers. The shielding is also designed as a set of cylindrical shells around the encapsulating source, though the size of this shielding may vary depending on the properties of the shielding materials. The second goal of this project is to determine the required thickness of a shield made of each individual material to meet the absorbed dose requirements, as well as creating additional shields designed to optimize for mass, cost, size, or a combination of these parameters.

The scope of this project is limited only to an exponential decay attenuation mathematical analysis of the specified source and shielding and an MCNP simulation of the encapsulation and source. The source is not modelled in MCNP with shielding, only with encapsulation, and the mathematics used to analyze the shielding do not consider scattered photons, buildup, or other secondary effects.

2. Methods

2.1. Problem Geometry and Parameters

The Cobalt-60 source used in this project has an activity of 1 Ci uniformly distributed over a cylindrical volume with a radius of 2.5 cm and a height of 10 cm. Cobalt-60 emits two high-energy gamma rays with energies of 1.17 MeV and 1.33 MeV respectively; both photons are emitted with functionally 100% frequency per decay. The inner encapsulation layer consists of 5 mm of stainless steel, the middle layer of 2 cm of either ceramic or graphite, and the outer layer of 1 cm of lead. Note that thickness in this case refers to both the radial and height directions; each encapsulation layer takes the form of a hollow cylindrical shell that completely surrounds all previous encapsulation layers, as well as the source itself.

Additional shielding layer[s] may be applied to this set of encapsulation layers. These layers take the same hollow cylindrical shape as the encapsulation layers, and the thickness of each layer is altered in order to meet the provided absorbed dose requirements: 0.1 mSv/hr at a distance of 2 meters from the center of the source. These shielding layers may consist of any combination of lead, stainless steel, and concrete. In the case that multiple materials are used for a given shielding design, each is given its own distinct layer, i.e. the materials are not mixed into a single compound material layer.

2.2. Encapsulation Simulation

MCNP is used here as a Monte-Carlo tool which can simulate the physical interactions of Cobalt-60's photons and the encapsulation layers. The source and encapsulation geometry described above was modelled using a combination of cylindrical and plane surface cards to generate cylindrical cell cards (see Figure 1); logical operators were used to create hollow voids, rather than # operators, which reduce MCNP performance (Shultis & Bahadori, 2024) (Los Alamos National Laboratory, 2022). The MCNP simulation was run with both ceramic and graphite encapsulation middle layers for 1,000,000 particle histories. Additional shielding geometry was analyzed using other techniques and is therefore not included in the MCNP analysis. A sphere of air with a radius of two meters surrounds the source and encapsulation layers, and a particle graveyard exists everywhere outside of this sphere to terminate particles once they leave the project's region of interest (see Figure 2). An F5 point detector, a fluence tally (Shultis & Bahadori, 2024, 21-22), is placed on the edge of this air sphere in the same plane as the radii of the source and encapsulation cylinders. The tally has a spherical exclusion zone of 5 cm to prevent particles from being oversampled or causing singularities by passing too close to the exact location of the tally. The mean tally value can then be converted from detections/cm^2/decay to mSv/hr as follows (Tanaka & Suzuki, 1991, 18):

$$X \frac{particles}{cm^2 * decay} \times 3.7 \cdot 10^{10} \frac{decays}{s} \times (1.172 + 1.333) \frac{MeV}{particle} \times 1.82 \cdot 10^{-8} \frac{Sv * cm^2 * s}{hr} \times 10^3 \frac{mSv}{Sv}$$

The resulting value has units of mSv/hr, as required in the project statement.

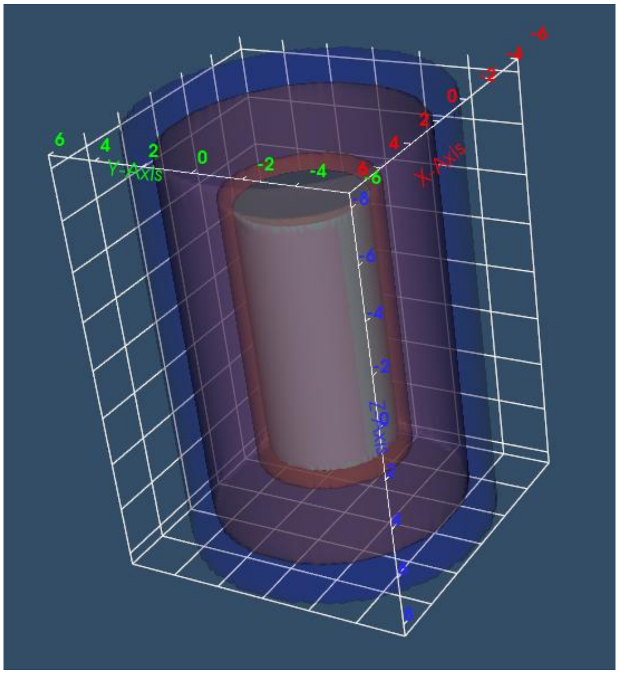


Figure 1: The Cobalt-60 source (center cylinder) surrounded by the appropriate encapsulation layers, as specified in MCNP.

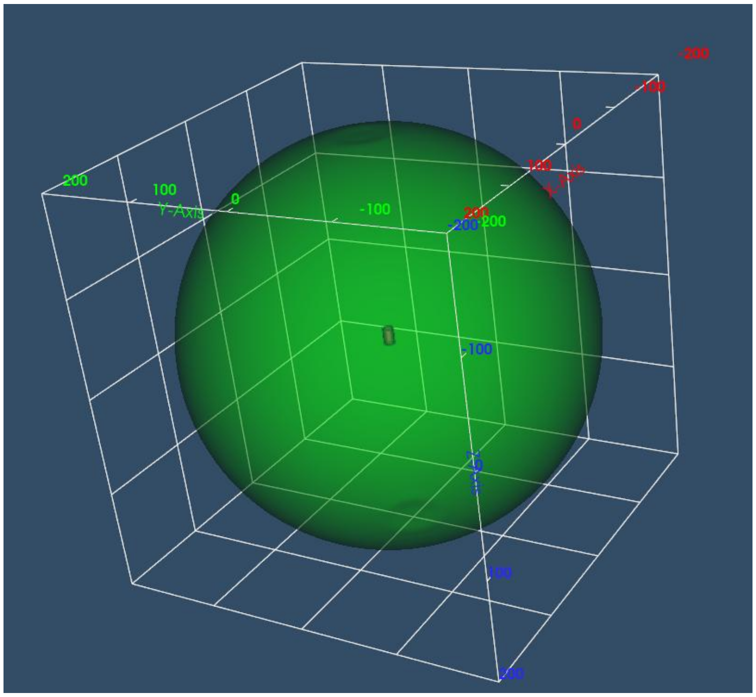


Figure 2: The Cobalt-60 source and encapsulation layers surrounded by a two-meter radius sphere of air, as specified in MCNP.

2.3. Encapsulation Mathematics

The dose rate $\dot{X}$ due to Cobalt-60 can be calculated using its specific gamma ray constant Γ (R-cm^2/hr mCi), activity A (mCi), and the distance from the source r (cm):

$$\dot{X} = \frac{\Gamma A}{r^2}$$

This equation gives $\dot{X}$ in Roentgens/hr, which must be converted to mSv/hr to be compared to the specifications of the project guidelines:

$$1\ R = 0.00877\ Sv = 8.77\ mSv$$

The impact of encapsulation on attenuating the source's radiation can then be modeled using exponential decay. Each encapsulation material has an attenuation coefficient μ (1/cm) and thickness t which can be used to determine the reduction it will have on dose rate as follows:

$$I = I_0 \times e^{-\mu t}$$

Where I_0 is the dose rate in mSv/hr with no encapsulation layers, and I is the dose rate in mSv/hr with encapsulation layers. This calculation can be sequentially performed for each layer of encapsulation to determine the total effect of all encapsulation layers on the radiation output by Cobalt-60.

The difference in performance between ceramic and graphite encapsulation can also be mathematically analyzed. Since the thickness of the middle encapsulation layer is required to be 2 cm regardless of the material used, simply changing the attenuation constant in the above equation can generate absorbed dose values for both forms of encapsulation. These results can then be compared to MCNP's results to verify this mathematical technique, allowing it to be used for shielding analysis with confidence.

2.4. Single Material Shielding Analysis

The equations used to analyze the encapsulation layer can be generalized to analyze additional shielding layers as well. In the case of a single shielding material, the attenuation equation can simply be rearranged to solve for shielding thickness given the required dose rate of 0.1 mSv/hr:

$$t\ (cm) = \frac{ln(I/0.1\ mSv/hr)}{\mu\ (1/cm)}$$

Where t is the thickness of the external shielding layer, I is the dose rate of radiation after passing through the encapsulation layers only, and μ is the attenuation coefficient of the shielding material.

2.5. Multiple Material Shielding Analysis

A more advanced mathematical analysis is required to solve for the required thickness of multiple shielding layers simultaneously. The equation used for the attenuation of these layers is as follows:

$$0.1\ mSv/hr = I \times e^{-\mu_1 t_1 - \mu_2 t_2 - \mu_3 t_3}$$

Where μ_i refers to the attenuation coefficient of the ith layer of material, and t_i refers to the thickness of the ith layer of material. This equation alone is not sufficient to derive a single numerical solution for thickness values, as many valid combinations of layer thickness exist depending on the order and ratio of materials used. To generate equations with a single solution, we chose to restrict each given problem to have a specified volume ratio of materials and tested many ratios in searching for the most optimal shielding combinations. The volume of a given shielding layer can be found as follows:

$$V = \pi(h_b r_b{}^2 - h_a r_a{}^2)$$

Where h_a and r_a refer to the height and radius of the hollow cylindrical void within the shielding layer (containing other shields and encapsulation), and h_b and r_b refer to the height and radius of the external shielding layer being analyzed. Since the shells have a consistent thickness across the radii and height, h_b can be expressed in terms of the other variables:

$$V = \pi(r_b{}^2(h_a + 2(r_b - r_a)) - r_a{}^2 h_a)$$

This volume equation can be used to set fixed ratios for different shielding layers in terms of their volume. Our model sets a fixed proportion for the volume of the inner and middle layers respectively:

$$p_1 = \frac{V_1}{V_T} \qquad p_2 = \frac{V_2}{V_T}$$

Here X is the proportion of volume that the inner layer consists of as compared to the full shielding volume, and Y is the same for the middle layer. V_1 is the volume of the inner layer, V_2 is the volume of the middle layer, and V_T is the volume of the entire shielding. The outer layer's volume proportion is then implicitly 1-X-Y. Rewriting the attenuation equation in terms of layer radius rather than layer thickness, and simplifying all equations shown here, gives the following system:

$$ln(I/0.1\ mSv/hr) = \mu_1(r_1 - r_0) + \mu_2(r_2 - r_1) + \mu_3(r_3 - r_2)$$

$$p_2 = \frac{{r_1}^2(h_0 + 2(r_1 - r_0)) - {r_0}^2 h_0}{{r_3}^2(h_0 + 2(r_3 - r_0)) - {r_0}^2 h_0}$$

$$p_2 = \frac{{r_2}^2(h_0 + 2(r_2 - r_0)) - {r_1}^2(h_0 + 2(r_1 - r_0))}{{r_3}^2(h_0 + 2(r_3 - r_0)) - {r_0}^2 h_0}$$

Where r_0 and h_0 are the radius and height of the encapsulation layers, r_1 and h_1 are the parameters of the inner layer, r_2 and h_2 are the parameters of the middle layer, and r_3 and h_3 are the parameters of the outer layer. Since all variables aside from r_1, r_2, and r_3 are constant, and the equations are linearly independent, this system of equations will have at most one solution. This system of equations was then algorithmically solved for every possible order of material layers and for every possible set of volume proportions to one decimal place (i.e. $p_1 = 0, 0.1, 0.2\ldots$) using Scipy's fsolve, a numerical methods optimization function in Python (The SciPy community, 2024).

Once the required radii were determined for all possible shielding combinations, they can be optimized along any number of given parameters. We decided to focus on three parameters. The first is total mass of shielding, found by multiplying the required volume of each layer by its material density. The second is the total cost of shielding, similarly found by multiplying each layer's mass by its material cost per unit mass. The final is a custom "efficiency" parameter developed to attempt to combine the effects of mass and cost into a single parameter. This parameter was calculated by min-max normalizing the total mass and total cost of each shield design, then summing the two values. This operation effectively calculates the percentile of mass and cost for each design and sums them. For example, a shield with a total mass in the 50th percentile of all possible shield design masses, and 70th percentile total cost, would have an efficiency metric of 0.5+0.7=1.2. An optimal design would aim to minimize this parameter.

3. Results and Analysis

3.1. MCNP Results

MCNP's F5 tally provided the values for photon fluence at a distance of two meters for both ceramic and graphite middle layers, which are summarized in Table 1:

Table 1: MCNP F5 tally results and calculated dose rate

Middle Layer	F5 Tally Mean	F5 Tally Error	Dose Rate (mSv/hr)
Graphite	4.8981E-10	0.0863	0.8262
Ceramic	4.0496E-10	0.0789	0.6831

3.2. Source and Encapsulation Analysis

Running the mathematical analysis described in Section 2.3 with the material parameters specified in Appendix A generated a dose rate of 2.895 mSv/hr from the source with no encapsulation layers. The results with encapsulation layers are described in Table 2:

Table 2: Basic Source and Encapsulation Dose Rates at a Distance of Two Meters

Encapsulation Layers	Dose Rate (mSv/hr)	Absolute % Difference from MCNP
Steel, Graphite, Lead	0.8048	2.59%
Steel, Ceramic Lead	0.6722	1.59%

MCNP's simulated dose rate values were slightly higher than our predicted values, likely due to simulated secondary effects such as backscattering or build up in the encapsulation layers. However, these error margins are below a 5% significance threshold and therefore verify that our mathematical models agree with simulation data, allowing them to be used for further shielding analysis. Both models agree that ceramic is a more effective encapsulation material than graphite in terms of dose rate reduction. See Appendix B for detailed calculations.

3.3. Single Material Shield Analysis

The required radius for a shield of each shielding material was determined for each combination of encapsulation layers using the calculations established in Section 2.4. The results of these calculations are summarized in Table 3.

Table 3: Single Material Shield Thickness Values

Encapsulation Layers	Shielding Material	Required Thickness (cm)
Steel, Graphite, Lead	Regulation Concrete	15.33
	Stainless Steel 304	4.534
	Lead	2.708
Steel, Ceramic, Lead	Regulation Concrete	14.011
	Stainless Steel 304	4.142
	Lead	2.475

Lead is the most effective shielding material in terms of size due to its large attenuation coefficient. These results also verify that for all types of additional shielding material simulated, the use of ceramic as the middle layer in the initial shielding configuration is the optimal choice over graphite. Ceramic is able to reduce dose rate more than graphite, so less shielding is required. The most optimal single material shield combination is therefore a lead outer shield with a ceramic middle encapsulation layer.

3.4. Multiple Material Shield Analysis

As ceramic was found to be the most optimal encapsulation layer, the steel-ceramic-lead initial dose rate of 0.6722 mSv/hr was used for further shield optimization analysis. The most optimal shield compositions are summarized in Table 4. A column that lists "-" indicates that the optimal shielding configuration only requires one or two layers, and that the second or third material[s] are not used in the optimized design. A ternary plot including the most optimal shielding configuration for each parameter is also included below.

Table 4: Most Optimal Shielding Configurations

<table>
<tr><th colspan="2">Metric</th><th>Total Mass (kg)</th><th>Total Cost (USD)</th><th>Efficiency Metric</th></tr>
<tr><td colspan="2">Metric Value</td><td>34.357</td><td>$9.661</td><td>0.287</td></tr>
<tr><td rowspan="2">Inner Layer</td><td>Material</td><td>Lead</td><td>Concrete</td><td>Lead</td></tr>
<tr><td>Thickness (cm)</td><td>2.475</td><td>14.011</td><td>1.992</td></tr>
<tr><td rowspan="2">Middle Layer</td><td>Material</td><td rowspan="4">-</td><td rowspan="4">-</td><td>Concrete</td></tr>
<tr><td>Thickness (cm)</td><td>2.730</td></tr>
<tr><td rowspan="2">Outer Layer</td><td>Material</td><td rowspan="2">-</td></tr>
<tr><td>Thickness (cm)</td></tr>
</table>

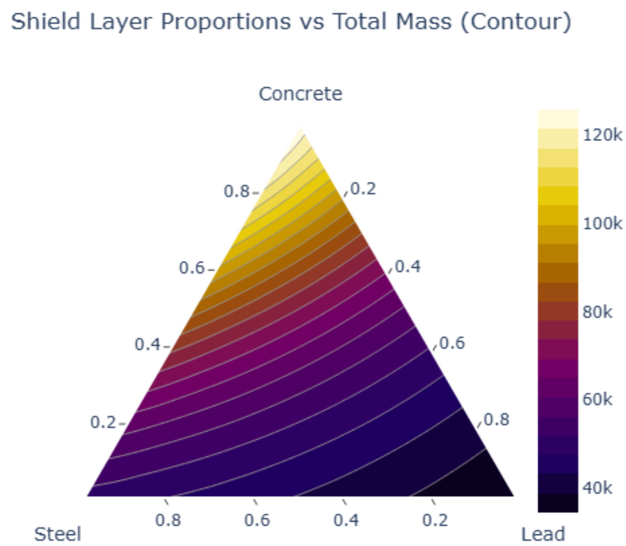


Figure 3: Shield layer proportions compared to mass (in grams). Here the inner layer is concrete, the middle layer is steel, and the outer layer is lead. Note the low mass as the proportion of lead increases.

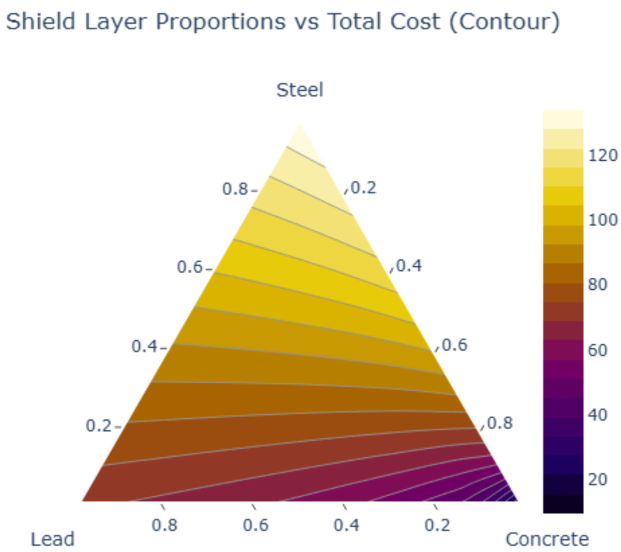


Figure 4: Shield layer proportions compared to cost (in USD). Here the inner layer is steel, the middle layer is lead, and the outer layer is concrete. Note the low cost as the proportion of concrete increases.

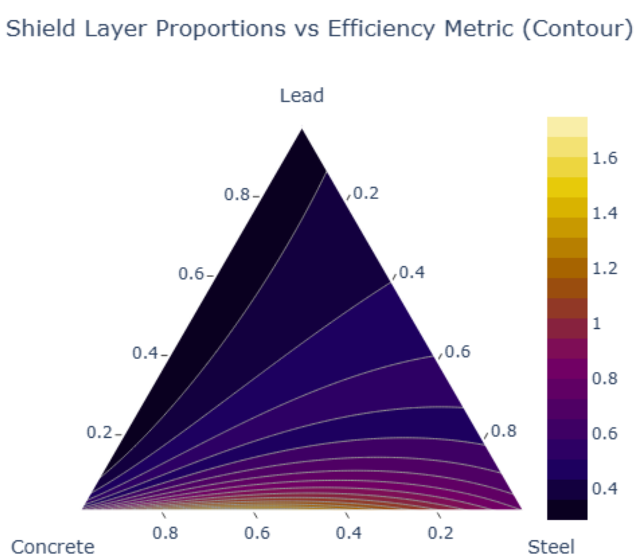


Figure 5: Shield layer proportions compared to the custom efficiency metric. Here the inner layer is lead, the middle layer is concrete, and the outer layer is steel. Note the low cost in the region where lead and concrete make up most or all the shield.

An inner lead (30% volume) and outer concrete (70% volume) shield is most optimal with respect to the custom efficiency metric; the design specified here is within the bottom 5% of all possible shield masses and bottom 25% of shield prices. Notably, stainless steel is not included in any of the most optimized shields.

The least optimal shield combinations were also calculated, and these results are summarized in Table 5:

Table 5: Least Optimal Shielding Configurations

<table>
<tr><th colspan="2">Metric</th><th>Total Mass (kg)</th><th>Total Cost (USD)</th><th>Efficiency Metric</th></tr>
<tr><td colspan="2">Metric Value</td><td>125.85</td><td>$188.78</td><td>1.76</td></tr>
<tr><td rowspan="2">Inner Layer</td><td>Material</td><td>Concrete</td><td>Concrete</td><td>Concrete</td></tr>
<tr><td>Thickness (cm)</td><td>14.011</td><td>6.578</td><td>8.166</td></tr>
<tr><td rowspan="2">Middle Layer</td><td>Material</td><td rowspan="4">-</td><td>Stainless Steel</td><td>Stainless Steel</td></tr>
<tr><td>Thickness (cm)</td><td>2.177</td><td>1.728</td></tr>
<tr><td rowspan="2">Outer Layer</td><td>Material</td><td rowspan="2">-</td><td rowspan="2">-</td></tr>
<tr><td>Thickness (cm)</td></tr>
</table>

The most optimal shield design in terms of mass is roughly 3.6 times lighter than the least optimal design, and the most cost-efficient design is nearly 20 times cheaper than the least cost effective. The shielding design that scores worst on the efficiency metric is in the top 20% of shield masses and top 6% of shield prices.

4. Discussion

The results of the team’s analysis demonstrate via both mathematical analysis and MCNP simulations that ceramic is the optimal material for the middle encapsulation layer. This is because it significantly reduces the dose rate compared to graphite. Ceramic as the middle layer offers a practical solution by keeping the shielding durable and lightweight while also increasing the gamma-ray attenuation efficiency. With this used as the material, the shielding configuration is improved in minimizing radiation exposure to the required acceptable levels (0.1 mSv/hr at 2 meters).

Lead is overall the most effective individual shielding material. Lead's extremely large attenuation coefficient when compared to stainless steel and concrete outweighs its large density under most analyses; a pure lead shield is most optimal with respect to both shield mass and shield volume. It provides exceptional performance in minimizing the thickness and mass of the shield and material use and could be used in future situations that require dense shielding solutions. Since the outer encapsulation layer also consists of lead, using lead as a shield is also physically convenient, as it would simply require extending the encapsulation, rather than adding a new material layer.

Concrete's extremely low cost and density when compared to stainless steel and lead outperforms its small attenuation coefficient when it comes to shield price, so a pure concrete shield is most optimal in terms of total cost. It combines well as an outer layer alongside an inner layer of lead; a shielding combination of 30% lead by volume and 70% concreted by volume performs well across all relevant metrics. An inner layer of concrete, on the other hand, performs very poorly; its low cost is best spent on the outer layers, where an increase in thickness requires more material proportionally than an inner layer does.

Steel's high cost, somewhat large density, and mediocre attenuation coefficient caused it to perform poorly on most optimization metrics. Unless the material properties of stainless steel are desired for the external shielding layer, it is generally not advisable to use steel as a radiation shield if one chooses to optimize for mass, volume, or cost, or any combination of the three.

Looking at shielding with multiple materials helps versatile planning to balance factors such as size, mass, and cost with spatial constraints. The process used in this report could be applied to other areas in industries dealing with the transportation of radioactive materials, medical facilities, or nuclear power plants. The results of the MCNP simulations provide value alongside mathematical models for insight into material interactions and secondary effects. By using both, an individual can ensure that the designs for shielding are viable and theoretically sound. The findings of this project show the importance of configuration and material selection when making efficient radiation shielding systems, creating change for future advancements in efficiency and safety in other applications.

5. Conclusion and Recommendations

As shown above, the project successfully analyzed a design for an optimized shielding system for the Cobalt-60 volume source. Ceramic was proven to be a more effective encapsulation material than graphite for the middle layer because of the dose rate reduction. Lead was the most effective single shielding material due to its high attenuation coefficient and density of the material. A combination of lead and concrete shielding can outperform a pure lead shield on a broader set of metrics.

In future work, the shielding results mathematically derived in this paper could be verified using MCNP. Using MCNP to simulate these material shielding layers would specifically aid in further verifying the mathematical models applied here and can more effectively account for build-up or scattering. These results could also be expanded by using volume proportions with a step size smaller than 0.1. By incorporating more granular volume ratios in shielding designs for multiple materials, the optimization process could be explored to find even more optimal shielding designs. Additional shielding materials could also be explored, such as tungsten or cadmium, materials with a similar density to lead. Finally, these mathematical and simulation techniques could also be applied to other radiation sources and geometries that more closely reflect actual physical reactor designs. With these recommendations, the models established here could be further generalized to industry or research reactors or further refined to solve stated problems in a more precise manner.

References

Hubbell, J. H., & Seltzer, S. M. (2004). Tables of X-Ray Mass Attenuation Coefficients and Mass Energy-Absorption Coefficients from 1 keV to 20 MeV for Elements Z = 1 to 92 and 48 Additional Substances of Dosimetric Interest. NIST Standard Reference Database, 126. NIST. https://dx.doi.org/10.18434/T4D01F

Los Alamos National Laboratory. (2022). *MCNP® Code Version 6.3.0 Theory & User Manual*. Los Alamos National Laboratory.

MIT. (2000). *Material Properties*. MIT. https://web.mit.edu/course/3/3.11/www/modules/props.pdf

The SciPy community. (2024). *fsolve — SciPy v1.14.1 Manual*. Numpy and Scipy Documentation. Retrieved December 5, 2024, from https://docs.scipy.org/doc/scipy/reference/generated/scipy.optimize.fsolve.html

ShowThePlanet Inc. (2024, December 5). *Daily Metal Spot Pricess*. Daily Metal Prices. https://www.dailymetalprice.com/metalprices.php?c=pb&u=oz&d=1

Shultis, J. K., & Bahadori, A. A. (2024, April 30). *An MCNP6 Primer* (1). Kansas State University. https://smartlab.mne.ksu.edu/static/jks/books.htm

Tanaka, S., & Suzuki, T. (1991, March). A CALCULATIONAL METHOD OF PHOTON DOSE EQUIVALENT BASED ON THE REVISED TECHNICAL STANDARDS OF RADIOLOGICAL PROTECTION LAW. https://www.osti.gov/servlets/purl/6197219

Tschudi, R., & HomeAdvisor. (2024, September 10). *How Much Does Concrete Cost Per Yard in 2024?* HomeAdvisor. Retrieved December 5, 2024, from https://www.homeadvisor.com/cost/outdoor-living/deliver-concrete/

Appendix A. Material Properties

Material	Density (g/cm^3)	Attenuation Coefficient (cm^{-1}) [1]	Cost (USD/kg)
Air	0.001205	-	-
Ceramic (Al_2O_3)	3.99	0.23	-
Cobalt-60	8.9	-	-
Graphite	2.26	0.14	-
Lead	11.34	0.77	$2.026 [2]
NRC Regulatory Concrete	2.3	0.136	$0.07677 [3]
Stainless Steel 304	7.93	0.46	$2.70 [4]

[1] (Hubbell & Seltzer, 2004)

[2] (ShowThePlanet Inc., 2024)

[3] (Tanaka & Suzuki, 1991)

[4] (MIT, 2000)

Appendix B. Notable Calculations

Cobalt-60 Raw Dose Rate at Two Meters:

$$\dot{X} = \frac{\Gamma A}{r^2} = \frac{13.2\ R - cm^2/hr - mCi\ *\ 1000mCi}{(200\ cm)^2} * 8.77\ mSv/R = 2.895\ mSv/hr$$

Cobalt-60 Dose Rate with Graphite Encapsulation:

$$I = 2.895mSv/hr \times e^{-0.46cm^{-1}*0.5\ cm - 0.14cm^{-1}*2\ cm - 0.77cm^{-1}*1\ cm} = 0.8048\ mSv/hr$$

Cobalt-60 Dose Rate with Ceramic Encapsulation:

$$I = 2.895mSv/hr \times e^{-0.46cm^{-1}*0.5\ cm - 0.23cm^{-1}*2\ cm - 0.77cm^{-1}*1\ cm} = 0.6722\ mSv/hr$$

Lead Shield Thickness with Ceramic Encapsulation:

$$t = \frac{ln(0.6722\ mSv/hr\ /\ 0.1\ mSv/hr)}{0.77\ (cm^{-1})} = 2.475\ cm$$

Appendix C. MCNP Input Deck

This input deck is redacted in compliance with US nuclear technology export controls.

Appendix D. Python Shielding Computation Code

```
# dict_keys(['Greys', 'YlGnBu', 'Greens', 'YlOrRd', 'Bluered', 'RdBu',
'Reds', 'Blues', 'Picnic', 'Rainbow', 'Portland', 'Jet', 'Hot', 'Blackbody',
'Earth', 'Electric', 'Viridis', 'Cividis'])

import math
import itertools
import numpy as np
from scipy.optimize import fsolve
import plotly.figure_factory as ff
import numpy as np
import pandas as pd
import os

if not os.path.exists("images"):
    os.mkdir("images")

# Dose mSv/hr
# I_0 = 0.8048151  # Graphite
I_0 = 0.672268  # Ceramic
I = 0.1
# Materials in CSG Units
lead = {'name': 'Lead', 'mu': 0.77, 'rho': 11.34, 'dollar_per_gram':
0.002026}
concrete = {'name': 'Concrete', 'mu': 0.136, 'rho': 2.3, 'dollar_per_gram':
0.00007677}
steel = {'name': 'Steel', 'mu': 0.46, 'rho': 7.93, 'dollar_per_gram': 0.0027}
material_list = [lead, concrete, steel]
# All orders of shielding material
material_sets = tuple(itertools.permutations(material_list))

# All permutations of proportion of inner and middle layers (by volume)
p_permutations = []
p1 = np.linspace(0, 1, 11)
for p in p1:
    p2 = np.linspace(0, 1, 11)
    for q in p2:
```

```
        p_set = [p, q, 1.0 - p - q]
        p_set = tuple([abs(round(elem, 1)) for elem in p_set])
        if (0 <= p_set[2] <= 1) and abs(sum(p_set) - 1.0) < 1e-10:
            p_permutations.append(p_set)
p_permutations = tuple(p_permutations)

# Inner radius and height
r_center = 6
h_center = 17

# Set of data can be found using
shielding_radii[mu_permutation][p_permutation]
# shielding_data = {}{perm0: {perm1: {'radii': [], 'volume': []} for perm1 in
p_permutations} for perm0 in mu_permutations}
shielding_data = {}

# Dataframes for ternary plot
df_layer1_data = pd.DataFrame({
    'name': [],
    'radius': [],
    'volume': [],
    'mass': [],
    'cost': [],
    'proportion': [],
})
df_layer2_data = pd.DataFrame({
    'name': [],
    'radius': [],
    'volume': [],
    'mass': [],
    'cost': [],
    'proportion': [],
})
df_layer3_data = pd.DataFrame({
    'name': [],
    'radius': [],
    'volume': [],
    'mass': [],
```

```
    'cost': [],
    'proportion': [],
})

layers_data = [df_layer1_data, df_layer2_data, df_layer3_data]

test_iter = 0;

for material_set in material_sets:
    for p_set in p_permutations:
        mu_inner = material_set[0]['mu']
        mu_middle = material_set[1]['mu']
        mu_outer = material_set[2]['mu']

        p_inner = p_set[0]
        p_middle = p_set[1]
        p_outer = p_set[2]


        # Define the system
        def system(vars):
            r1, r2, r3 = vars
            eq1 = ((((r1 ** 2) * (h_center + 2 * (r1 - r_center))) -
((r_center ** 2) * h_center)) / (
                    ((r3 ** 2) * (h_center + 2 * (r3 - r_center))) -
((r_center ** 2) * h_center))) - p_inner
            eq2 = ((((r2 ** 2) * (h_center + 2 * (r2 - r_center))) - ((r1 **
2) * (h_center + 2 * (r1 - r_center)))) / (
                    ((r3 ** 2) * (h_center + 2 * (r3 - r_center))) -
((r_center ** 2) * h_center))) - p_middle
            eq3 = ((np.log(I_0 / I) - (r1 * (mu_inner - mu_middle)) +
(r_center * mu_inner) - (mu_outer * r3)) / (
                        mu_middle - mu_outer)) - r2
            return [eq1, eq2, eq3]


        # Solve the system
        initial_guess = [8, 8, 8]
```

```
        solution = fsolve(system, initial_guess)

        # Solve for radii/volume of each layer and save
        r_inner = solution[0]
        r_middle = solution[1]
        r_outer = solution[2]
        r_set = [r_inner, r_middle, r_outer]
        r_set = [abs(round(elem, 3)) for elem in r_set]

        V_inner = math.pi * (((r_inner ** 2) * (h_center + 2 * (r_inner -
r_center))) - ((r_center ** 2) * h_center))
        V_middle = math.pi * (((r_middle ** 2) * (h_center + 2 * (r_middle -
r_center))) - (
                    (r_inner ** 2) * (h_center + 2 * (r_inner - r_center))))
        V_outer = math.pi * (((r_outer ** 2) * (h_center + 2 * (r_outer -
r_center))) - (
                    (r_middle ** 2) * (h_center + 2 * (r_middle -
r_center))))
        V_set = [V_inner, V_middle, V_outer]
        V_set = [abs(round(elem, 3)) for elem in V_set]

        for i in range(len(layers_data)):
            name = material_set[i]['name']
            radius = r_set[i]
            volume = V_set[i]
            mass = material_set[i]['rho'] * volume
            cost = material_set[i]['dollar_per_gram'] * mass
            proportion = p_set[i]
            new_row = [radius, volume, mass, cost, proportion]
            new_row = [abs(round(elem, 3)) for elem in new_row]
            new_row = [name] + new_row
            layers_data[i].loc[-1] = new_row
            layers_data[i].index = layers_data[i].index + 1
            layers_data[i] = layers_data[i].sort_index()

df_rename = []
for i, df in enumerate(layers_data, 1):
    new_columns = {
```

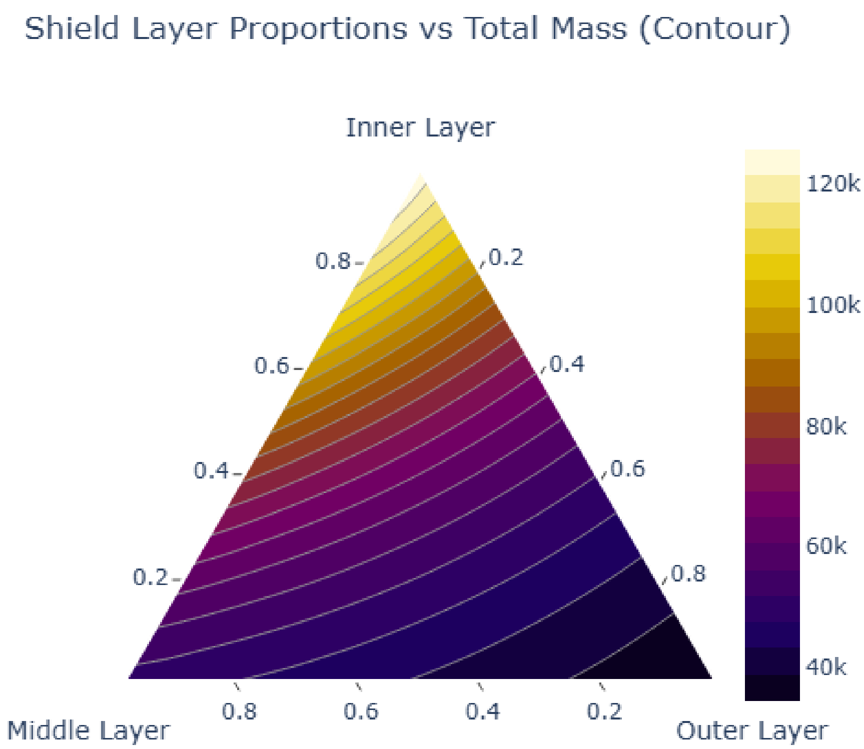
```
        'name': f'layer{i}_name',
        'radius': f'layer{i}_radius',
        'volume': f'layer{i}_volume',
        'mass': f'layer{i}_mass',
        'cost': f'layer{i}_cost',
        'proportion': f'layer{i}_proportion'
    }
    df_rename.append(df.rename(columns=new_columns))

# Combine all dataframes horizontally
df_shield_data = pd.concat(df_rename, axis=1)

# Add total mass/cost columns
df_shield_data['total_cost'] = (
        df_shield_data['layer1_cost'] +
        df_shield_data['layer2_cost'] +
        df_shield_data['layer3_cost']
)
df_shield_data['total_mass'] = (
        df_shield_data['layer1_mass'] +
        df_shield_data['layer2_mass'] +
        df_shield_data['layer3_mass']
)
df_shield_data['total_volume'] = (
        df_shield_data['layer1_volume'] +
        df_shield_data['layer2_volume'] +
        df_shield_data['layer3_volume']
)

# Try normalizing the cost/mass and making test product their multiple

df_shield_data['normal_mass'] = (df_shield_data['total_mass']-
df_shield_data['total_mass'].min())/(df_shield_data['total_mass'].max()-
df_shield_data['total_mass'].min())
df_shield_data['normal_cost'] = (df_shield_data['total_cost']-
df_shield_data['total_cost'].min())/(df_shield_data['total_cost'].max()-
df_shield_data['total_cost'].min())
```

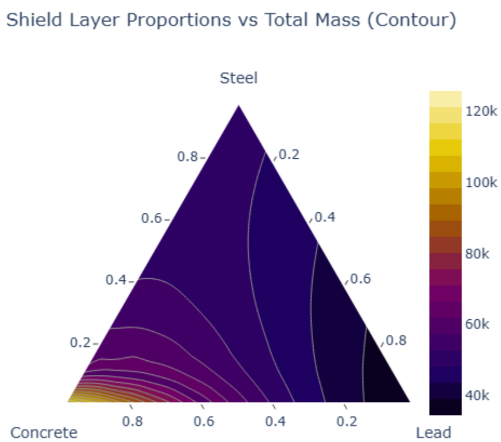
```
df_shield_data['combo_metric'] = df_shield_data['normal_mass'] +
df_shield_data['normal_cost']

check_vars = ['total_cost', 'total_mass', 'combo_metric']

for var in check_vars:
    min_value = df_shield_data[var].min()
    min_index = df_shield_data[[var]].idxmin()
    print("Minimum", str(var), "=", str(min_value), "\tIndex:",
str(min_index))
    print(df_shield_data.loc[min_index, :].to_string())
    print("")

    max_value = df_shield_data[var].max()
    max_index = df_shield_data[[var]].idxmax()
    print("Maximum", str(var), "=", str(max_value), "\tIndex:",
str(max_index))
    print(df_shield_data.loc[max_index, :].to_string())
    print("------------------------------------------------------------------
------------------------------------------")

'''
# Filter for rows where any layer is Steel with proportion 0.0
filtered_df = df_shield_data[
    ((df_shield_data['layer1_name'] == 'Steel') &
(df_shield_data['layer1_proportion'] == 0.0)) |
    ((df_shield_data['layer2_name'] == 'Steel') &
(df_shield_data['layer2_proportion'] == 0.0)) |
    ((df_shield_data['layer3_name'] == 'Steel') &
(df_shield_data['layer3_proportion'] == 0.0))
]
print(filtered_df.to_string())
'''

for i in range(len(material_sets)):
    data_length = int(len(df_shield_data)/len(material_sets))
    data_range = [int(data_length*i), int((data_length*(i+1))-1)]
```

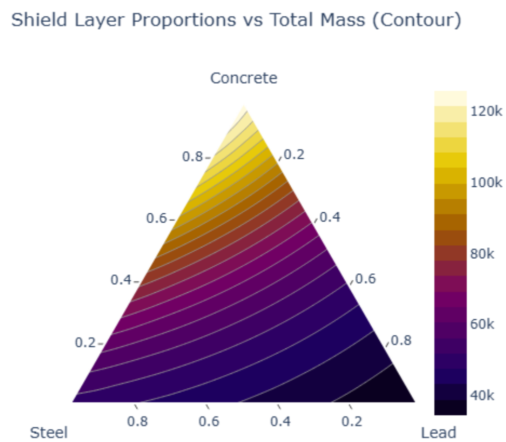
```
    df = df_shield_data.loc[data_range[0]:].head(data_length)
    prop1 = np.array(df['layer1_proportion'])
    prop2 = np.array(df['layer2_proportion'])
    prop3 = np.array(df['layer3_proportion'])
    name1 = df.iloc[0]['layer1_name']
    name2 = df.iloc[0]['layer2_name']
    name3 = df.iloc[0]['layer3_name']
    mass = np.array(df['total_mass'])
    cost = np.array(df['total_cost'])
    combo_metric = np.array(df['combo_metric'])

    # Top is layer 1, bottom left is layer 2, bottom right is layer 3
    fig = ff.create_ternary_contour(
        [prop1, prop2, prop3],
        combo_metric,
        pole_labels=[name1, name2, name3],
        interp_mode='cartesian',
        colorscale='Electric',
        title='Shield Layer Proportions vs Efficiency Metric (Contour)',
        showscale=True,
        ncontours=20
    )
    # fig.show()
"""
file_name = f"images/{name1}_{name2}_{name3}_mass.png"
print(file_name)
fig_mass.write_image(file_name, width=800, height=600)

filtered_df = df_shield_data[((df_shield_data['layer1_name'] == 'Steel') &
(df_shield_data['layer1_proportion'] == 1.0))]
print(filtered_df.head(1))
filtered_df = df_shield_data[((df_shield_data['layer1_name'] == 'Concrete') &
(df_shield_data['layer1_proportion'] == 1.0))]
print(filtered_df.head(1))
filtered_df = df_shield_data[((df_shield_data['layer1_name'] == 'Lead') &
(df_shield_data['layer1_proportion'] == 1.0))]
print(filtered_df.head(1))
"""
```

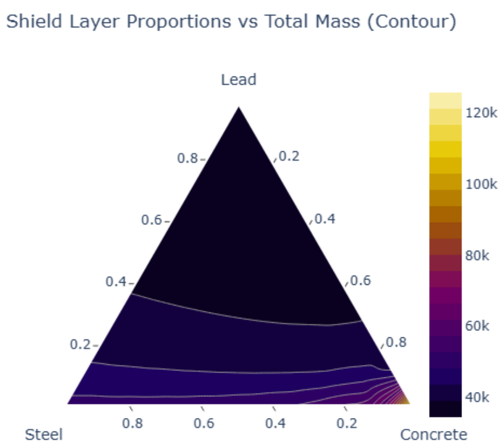
Appendix E. Ternary Plots for All Shielding Combinations

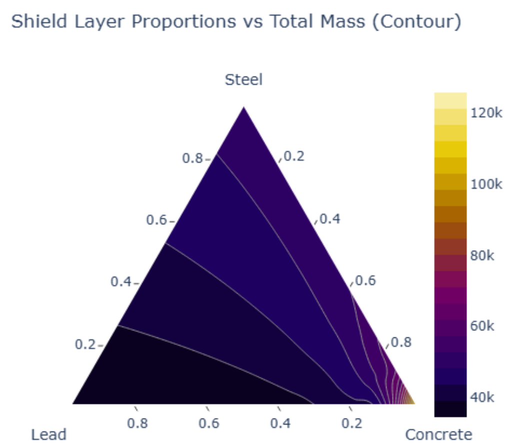
Note that for every plot, the innermost layer is the top material, the middle layer is the bottom left material, and the outermost player is the bottom right material. The units of mass are grams, and cost is displayed in USD. A sample ternary plot is displayed below for reference of which layer is inner, middle, and outer; this is followed by the plots for each relevant optimization metric.

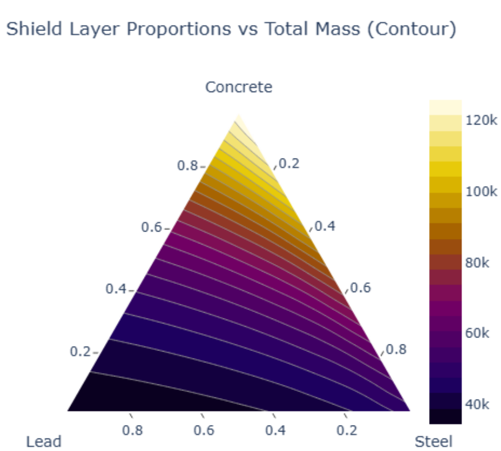


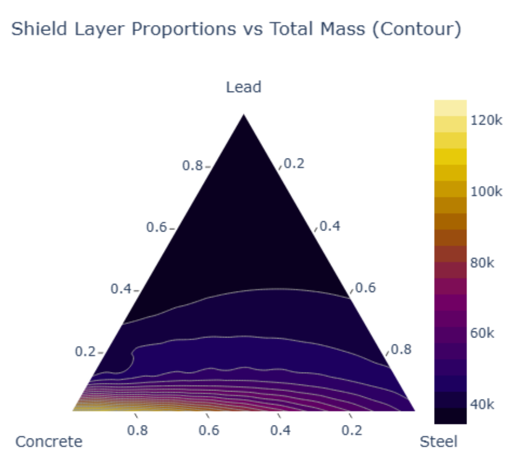

Shield Layer Proportions vs Total Mass (Contour)
Steel
Concrete
Lead
0.2
0.4
0.6
0.8
40k
60k
80k
100k
120k


Shield Layer Proportions vs Total Mass (Contour)
Concrete
Steel
Lead
0.2
0.4
0.6
0.8
40k
60k
80k
100k
120k

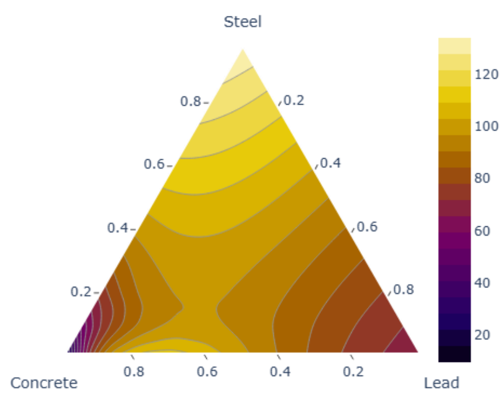

Shield Layer Proportions vs Total Mass (Contour)
Lead
Steel
Concrete
0.2
0.4
0.6
0.8
40k
60k
80k
100k
120k


Shield Layer Proportions vs Total Mass (Contour)
Steel
Lead
Concrete
0.2
0.4
0.6
0.8
40k
60k
80k
100k
120k

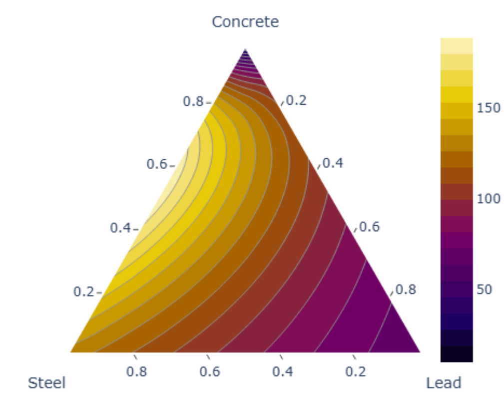

Shield Layer Proportions vs Total Mass (Contour)
Concrete
Lead
Steel
0.2
0.4
0.6
0.8
40k
60k
80k
100k
120k


Shield Layer Proportions vs Total Mass (Contour)
Lead
Concrete
Steel
0.2
0.4
0.6
0.8
40k
60k
80k
100k
120k

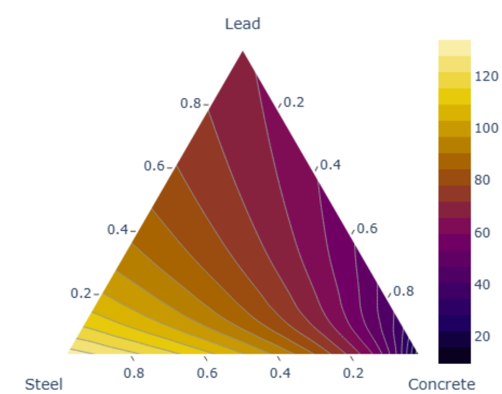
Shield Layer Proportions vs Total Cost (Contour)



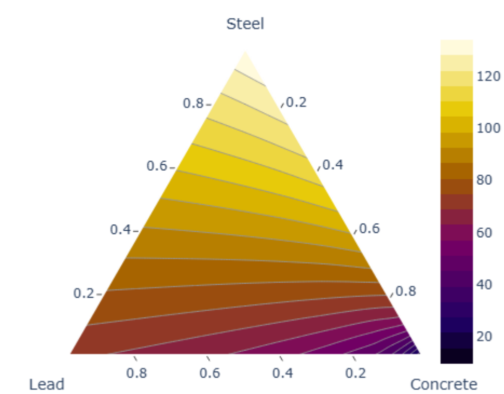
Shield Layer Proportions vs Total Cost (Contour)



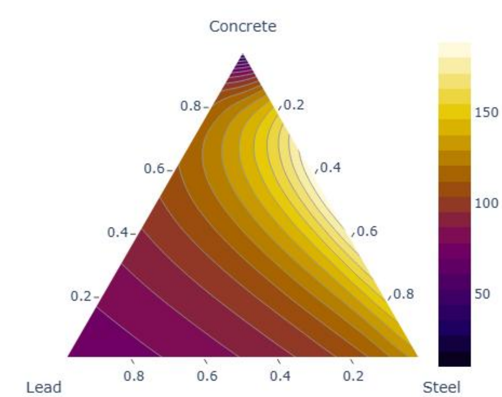
Shield Layer Proportions vs Total Cost (Contour)



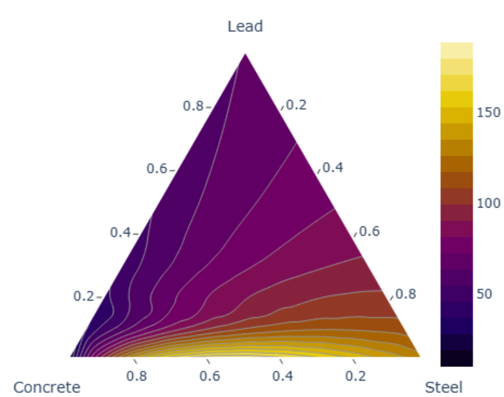
Shield Layer Proportions vs Total Cost (Contour)

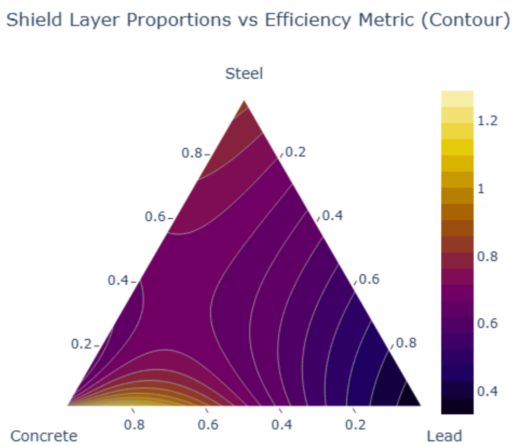


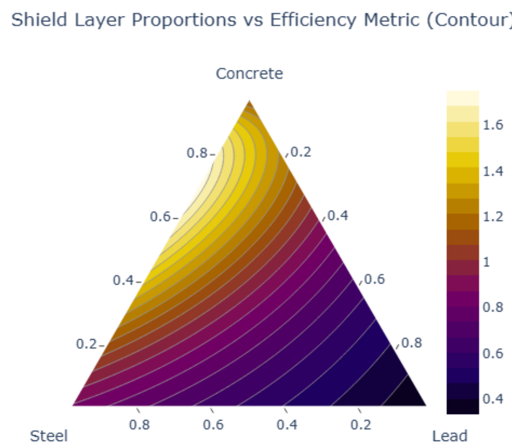
Shield Layer Proportions vs Total Cost (Contour)

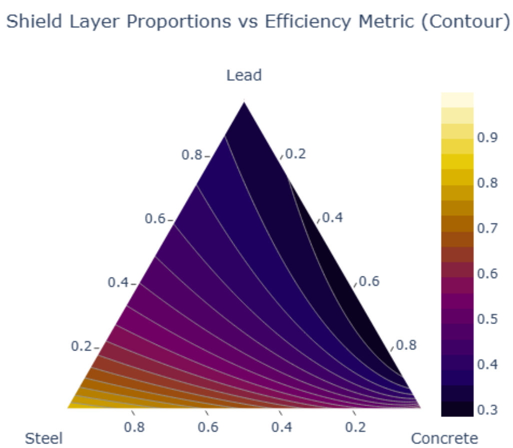


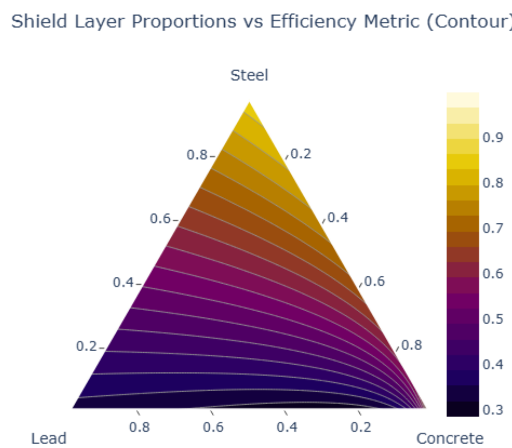
Shield Layer Proportions vs Total Cost (Contour)

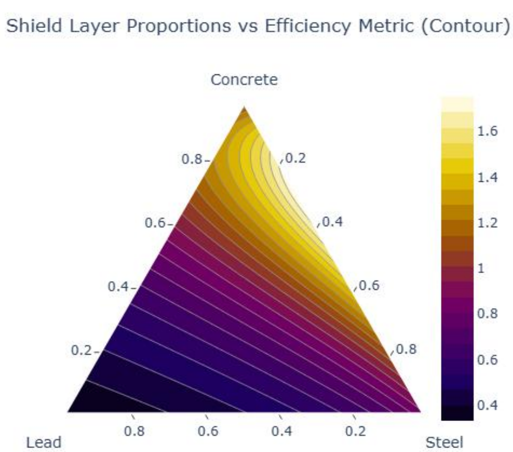


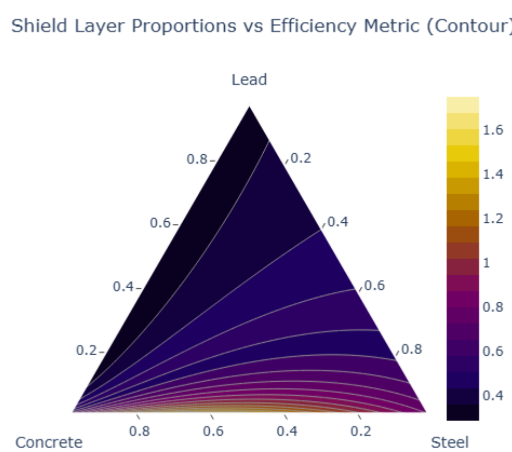

Shield Layer Proportions vs Efficiency Metric (Contour)
Steel
Concrete
Lead


Shield Layer Proportions vs Efficiency Metric (Contour)
Concrete
Steel
Lead


Shield Layer Proportions vs Efficiency Metric (Contour)
Lead
Steel
Concrete


Shield Layer Proportions vs Efficiency Metric (Contour)
Steel
Lead
Concrete


Shield Layer Proportions vs Efficiency Metric (Contour)
Concrete
Lead
Steel


Shield Layer Proportions vs Efficiency Metric (Contour)
Lead
Concrete
Steel